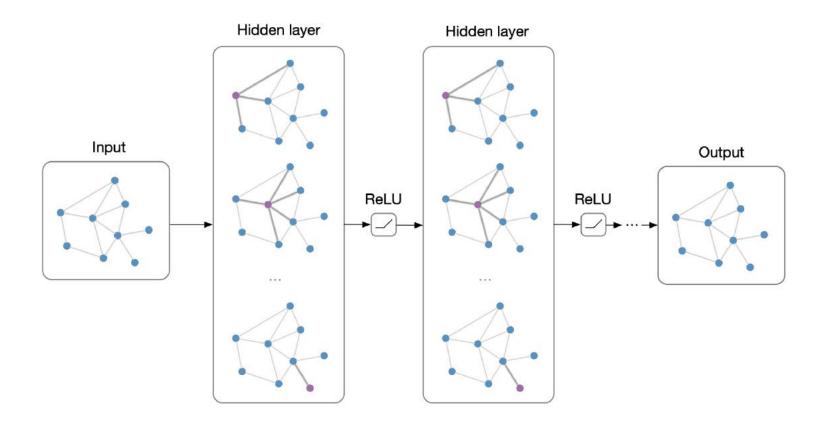
Graph Neural Networks



Xiachong Feng TG 2019-04-08

Relies heavily on

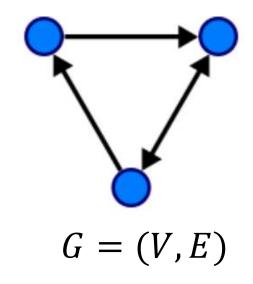
- A Gentle Introduction to Graph Neural Networks (Basics, DeepWalk, and GraphSage)
- Structured deep models: Deep learning on graphs and beyond
- Representation Learning on Networks
- Graph neural networks: Variations and applications
- http://snap.stanford.edu/proj/embeddings-www/
- Graph Neural Networks: A Review of Methods and Applications

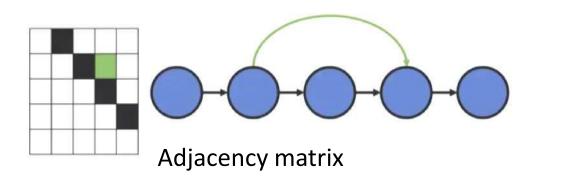
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Graph

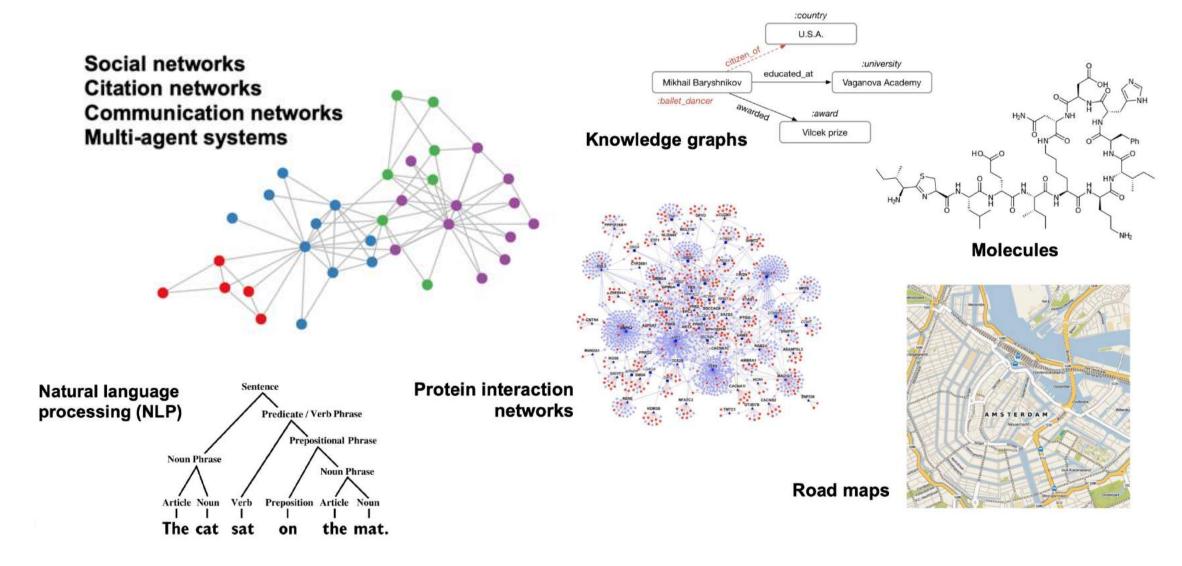
- Graph is a data structure consisting of two components, vertices and edges.
- A graph G can be well described by the set of vertices V and edges E it contains.
- Edges can be either directed or undirected, depending on whether there exist directional dependencies between vertices.
- The vertices are often called nodes. these two terms are interchangeable.



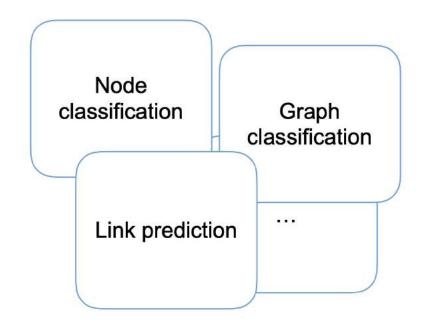


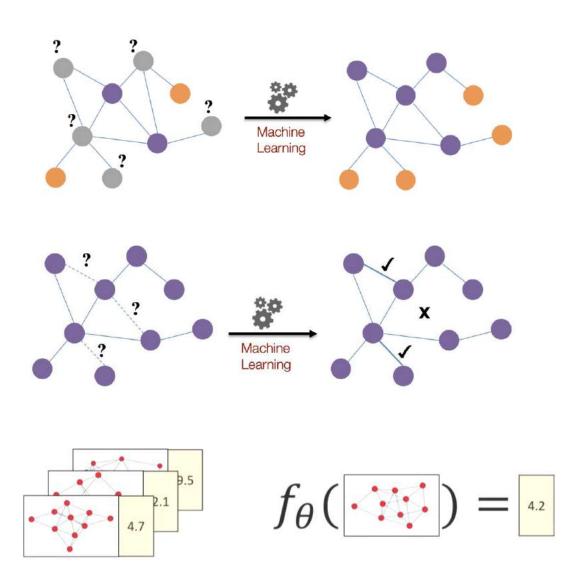
Graph structured data

Graph-Structured Data



Problems && Tasks

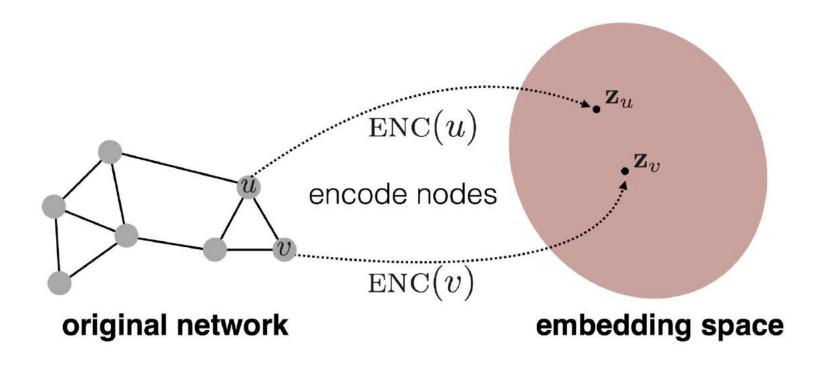




Representation Learning on Networks Graph neural networks: Variations and applications

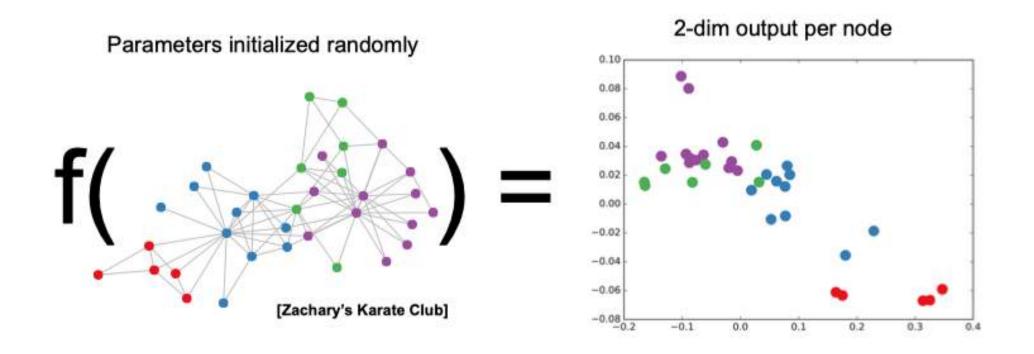
Embedding Nodes

• Goal is to encode nodes so that similarity in the embedding space (e.g., dot product) approximates similarity in the original network.



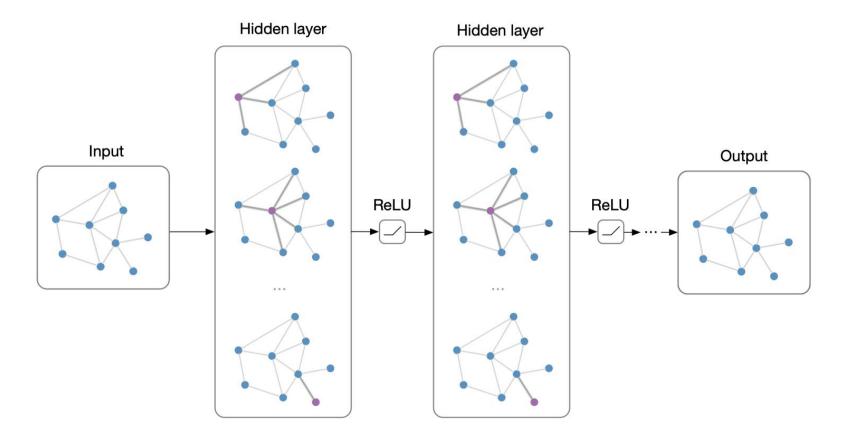
Embedding Nodes

 Graph Neural Network is a neural network architecture that learns embeddings of nodes in a graph by looking at its nearby nodes.



GNN Overview

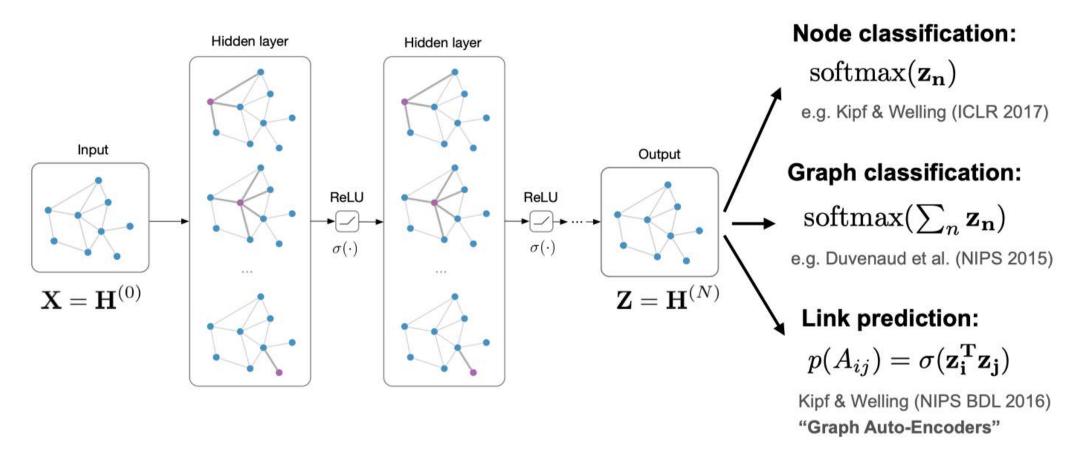
The bigger picture:



Main idea: Pass messages between pairs of nodes & agglomerate

GNN Overview

Input: Feature matrix $\mathbf{X} \in \mathbb{R}^{N imes E}$, preprocessed adjacency matrix $\hat{\mathbf{A}}$



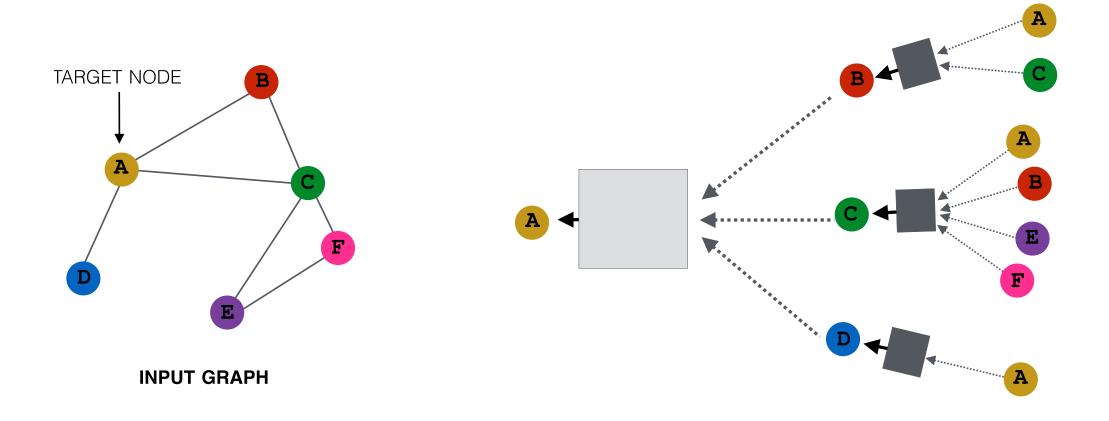
Why GNN?

- Firstly, the standard neural networks like CNNs and RNNs cannot handle the graph input properly in that they stack the feature of nodes by a specific order. To solve this problem, GNNs propagate on each node respectively, ignoring the input order of nodes.
- Secondly, GNNs can do propagation guided by the graph structure, Generally, GNNs update the hidden state of nodes by a weighted sum of the states of their neighborhood.
- Thirdly, reasoning. GNNs explore to generate the graph from nonstructural data like scene pictures and story documents, which can be a powerful neural model for further high-level AI.

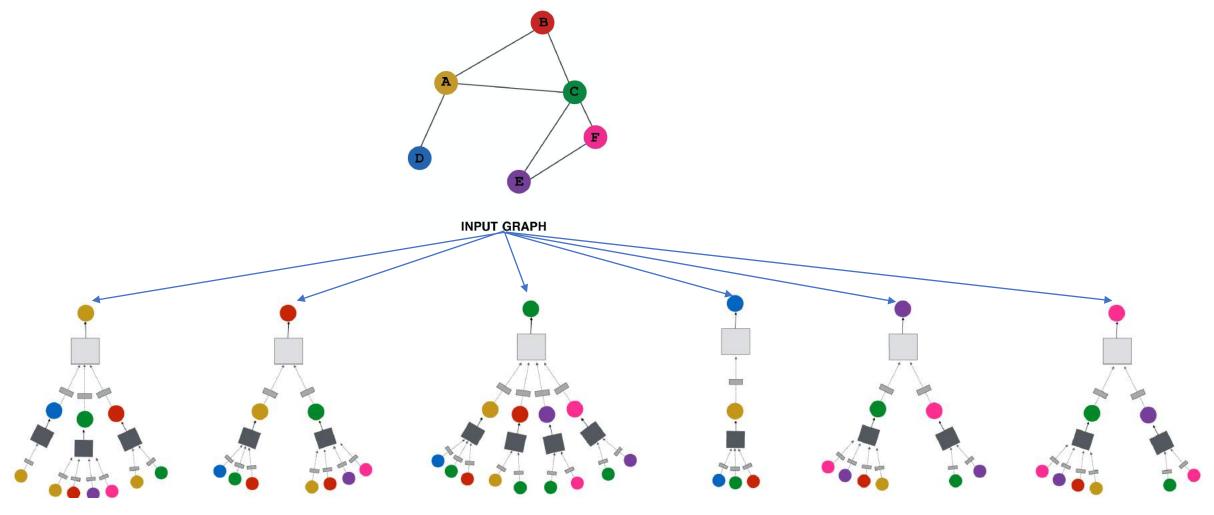
Outline

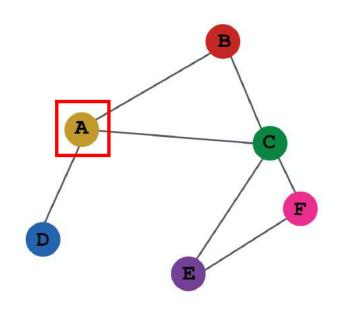
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- Key idea: Generate node embeddings based on local neighborhoods.
- Intuition: Nodes aggregate information from their neighbors using neural networks

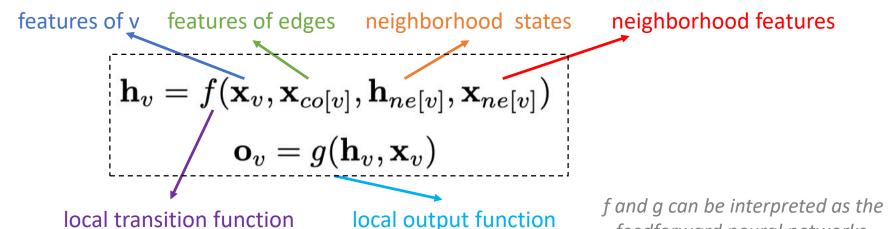


• Intuition: Network neighborhood defines a computation graph





INPUT GRAPH



$$\mathbf{H} = F(\mathbf{H}, \mathbf{X})$$

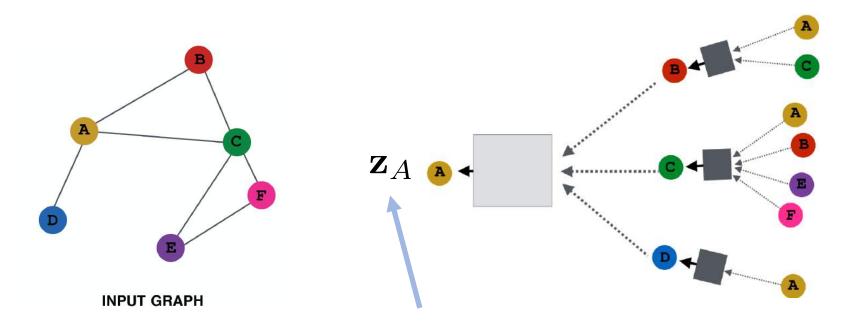
$$\mathbf{O} = G(\mathbf{H}, \mathbf{X}_N)$$

Banach's fixed point theorem

$$\mathbf{H}^{t+1} = F(\mathbf{H}^t, \mathbf{X})$$

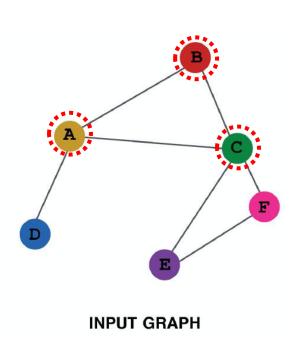
feedforward neural networks.

How do we train the model to generate high-quality embeddings?

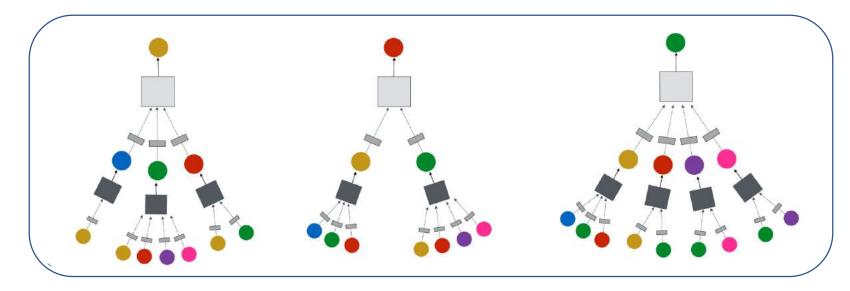


Need to define a loss function on the embeddings, L(z)!

• Train on a set of nodes, i.e., a batch of compute graphs



$$loss = \sum_{i=1}^p (\mathbf{t}_i - \mathbf{o}_i)$$



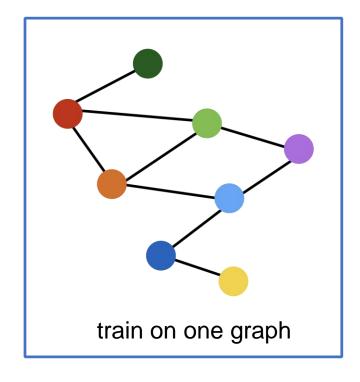
Gradient-descent strategy

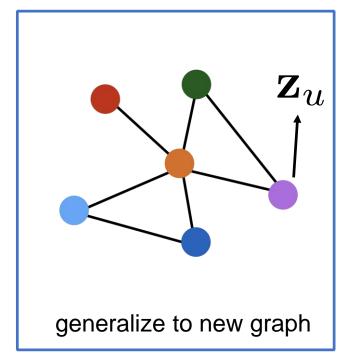
- The states h_v are iteratively updated by. $\mathbf{h}_v = f(\mathbf{x}_v, \mathbf{x}_{co[v]}, \mathbf{h}_{ne[v]}, \mathbf{x}_{ne[v]})$ a time T. They approach the fixed point solution of $H(T) \approx H$.
- The gradient of weights W is computed from the loss.
- The weights W are updated according to the gradient computed in the last step.

Inductive Capability

• Even for nodes we never trained on

- Inductive Capability
 - Inductive node embedding-->generalize to entirely unseen graphs



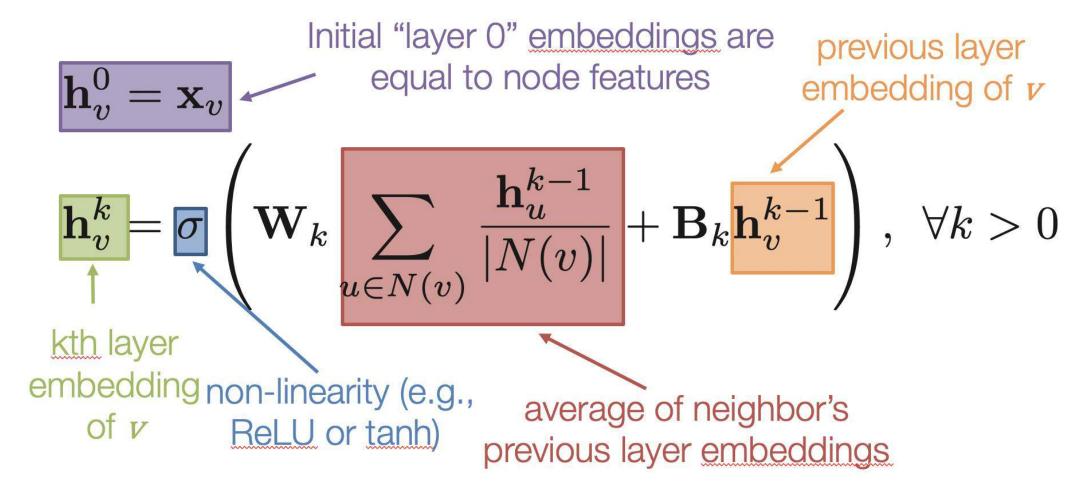


Limitations

- Firstly, it is inefficient to update the hidden states of nodes iteratively for the fixed point. If the assumption of fixed point is relaxed, it is possible to leverage Multi-layer Perceptron to learn a more stable representation, and removing the iterative update process. This is because, in the original proposal, different iterations use the same parameters of the transition function f, while the different parameters in different layers of MLP allow for hierarchical feature extraction.
- It cannot process edge information (e.g. different edges in a knowledge graph may indicate different relationship between nodes)
- Fixed point can discourage the diversification of node distribution, and thus may not be suitable for learning to represent nodes.

Average Neighbor Information

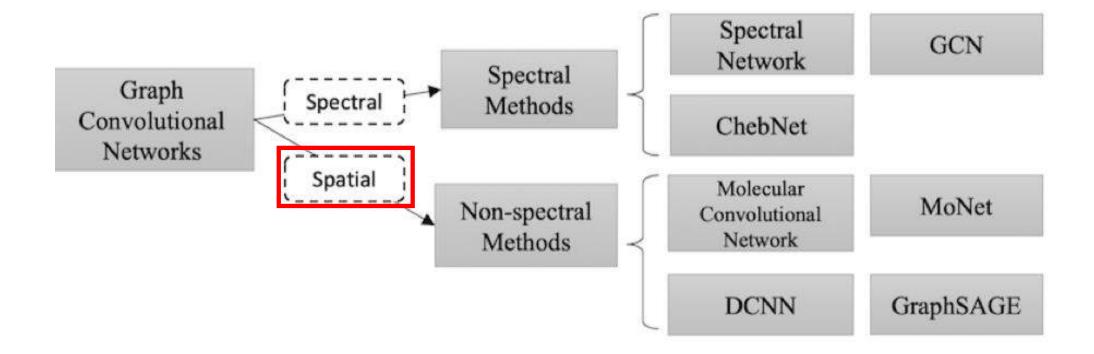
Basic approach: Average neighbor information and apply a neural network.



Outline

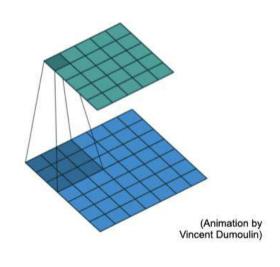
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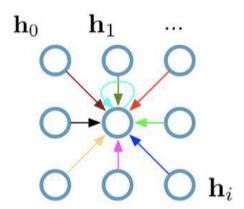
Graph Convolutional Networks (GCNs)



Convolutional Neural Networks (on grids)

Single CNN layer with 3x3 filter:





Update for a single pixel:

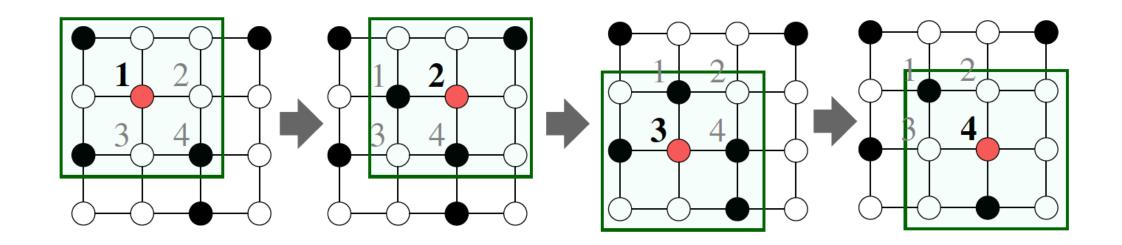
- Transform messages individually $\mathbf{W}_i\mathbf{h}_i$
- Add everything up $\sum_i \mathbf{W}_i \mathbf{h}_i$

 $\mathbf{h}_i \in \mathbb{R}^F$ are (hidden layer) activations of a pixel/node

Full update:

$$\mathbf{h}_{4}^{(l+1)} = \sigma \left(\mathbf{W}_{0}^{(l)} \mathbf{h}_{0}^{(l)} + \mathbf{W}_{1}^{(l)} \mathbf{h}_{1}^{(l)} + \dots + \mathbf{W}_{8}^{(l)} \mathbf{h}_{8}^{(l)} \right)$$

Convolutional Neural Networks (on grids)

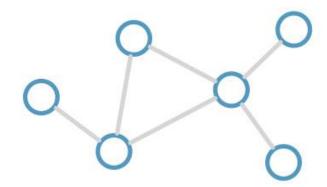


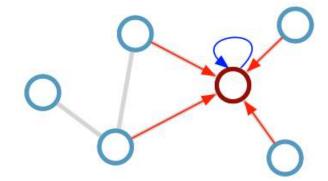
Graph Convolutional Networks (GCNs)

Consider this undirected graph:

Calculate update for node in red:

Convolutional networks on graphs for learning molecular fingerprints NIPS 2015





$$\mathbf{x} = \mathbf{h}_v + \sum_{i=1}^{\mathcal{N}_v} \mathbf{h}_i$$
 $\mathbf{h}_v' = \sigma ig(\mathbf{x} \mathbf{W}_L^{\mathcal{N}_v} ig)$

Update rule:

$$\mathbf{h}_{i}^{(l+1)} = \sigma \left(\mathbf{h}_{i}^{(l)} \mathbf{W}_{0}^{(l)} + \sum_{j \in \mathcal{N}_{i}} \frac{1}{c_{ij}} \mathbf{h}_{j}^{(l)} \mathbf{W}_{1}^{(l)} \right)$$

GraphSAGE

$$\mathbf{h}_{\mathcal{N}_v}^t = \text{AGGREGATE}_t \left(\left\{ \mathbf{h}_u^{t-1}, \forall u \in \mathcal{N}_v \right\} \right)$$
$$\mathbf{h}_v^t = \sigma \left(\mathbf{W}^t \cdot \left[\mathbf{h}_v^{t-1} || \mathbf{h}_{\mathcal{N}_v}^t \right] \right)$$

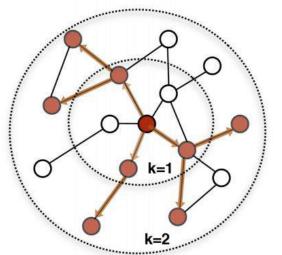
Mean aggregator.

$$\mathbf{h}_v^t = \sigma(\mathbf{W} \cdot \text{MEAN}(\{\mathbf{h}_v^{t-1}\} \cup \{\mathbf{h}_u^{t-1}, orall u \in \mathcal{N}_v\})$$
LSTM aggregator.

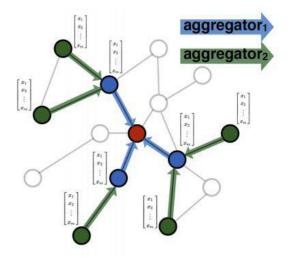
$$AGG = LSTM ([\mathbf{h}_u^{k-1}, \forall u \in \pi(N(v))])$$

Pooling aggregator.

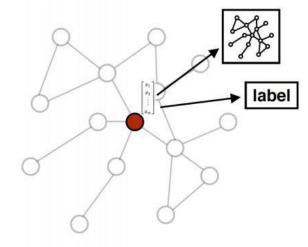
$$\mathbf{h}_{\mathcal{N}_v}^t = \max(\{\sigma\left(\mathbf{W}_{\text{pool}}\mathbf{h}_u^{t-1} + \mathbf{b}\right), \forall u \in \mathcal{N}_v\})$$



1. Sample neighborhood



2. Aggregate feature information from neighbors



3. Predict graph context and label using aggregated information

GraphSAGE

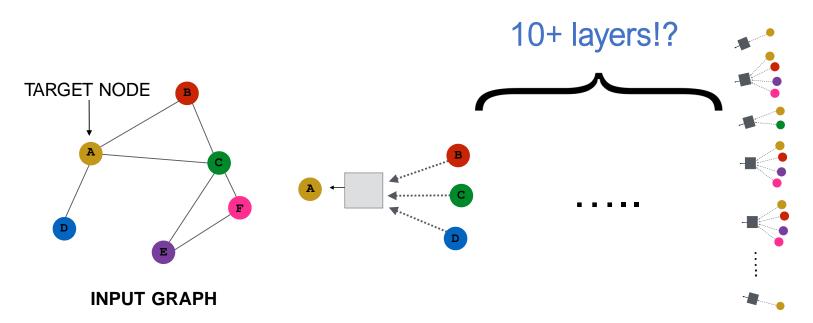
```
Algorithm 1: GraphSAGE embedding generation (i.e., forward propagation) algorithm
    Input: Graph \mathcal{G}(\mathcal{V}, \mathcal{E}); input features \{\mathbf{x}_v, \forall v \in \mathcal{V}\}; depth K; weight matrices
                   \mathbf{W}^k, \forall k \in \{1, ..., K\}; non-linearity \sigma; differentiable aggregator functions
                   AGGREGATE_k, \forall k \in \{1, ..., K\}; neighborhood function \mathcal{N}: v \to 2^{\mathcal{V}}
    Output: Vector representations \mathbf{z}_v for all v \in \mathcal{V}
1 \mathbf{h}_v^0 \leftarrow \mathbf{x}_v, \forall v \in \mathcal{V}; init
2 for k = 1...K do Kiters
          for v \in \mathcal{V} do For every node
       \mathbf{h}_{\mathcal{N}(v)}^k \leftarrow \text{AGGREGATE}_k(\{\mathbf{h}_u^{k-1}, \forall u \in \mathcal{N}(v)\}); \quad \textit{K-th func}
        \mathbf{h}_v^k \leftarrow \sigma\left(\mathbf{W}^k \cdot \text{CONCAT}(\mathbf{h}_v^{k-1}, \mathbf{h}_{\mathcal{N}(v)}^k)\right)
          end
       \mathbf{h}_v^k \leftarrow \mathbf{h}_v^k / \|\mathbf{h}_v^k\|_2, \forall v \in \mathcal{V}
8 end
9 \mathbf{z}_v \leftarrow \mathbf{h}_v^K, \forall v \in \mathcal{V}
```

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Gated Graph Neural Networks (GGNNs)

- GCNs and GraphSAGE generally only 2-3 layers deep.
- Challenges:
 - Overfitting from too many parameters.
 - Vanishing/exploding gradients during backpropagation.



Gated Graph Neural Networks (GGNNs)

- GGNNs can be seen as multi-layered GCNs where layer-wise parameters are tied and gating mechanisms are added.
 - 1. Get "message" from neighbors at step k:

$$\mathbf{m}_v^k = \mathbf{W} \sum_{u \in N(v)} \mathbf{h}_u^{k-1}$$
 aggregation function does not depend on \mathbf{k}

2. Update node "state" using Gated Recurrent Unit (GRU). New node state depends on the old state and the message from neighbors:

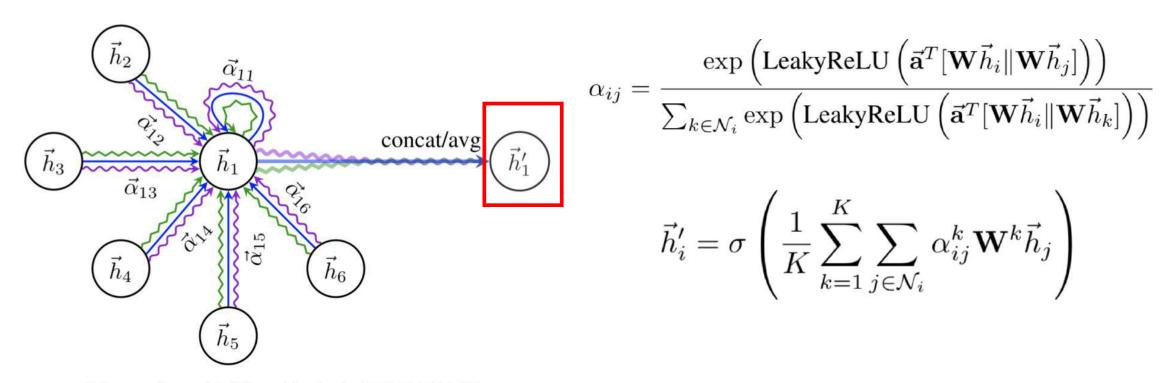
$$\mathbf{h}_{v}^{k} = \mathrm{GRU}(\mathbf{h}_{v}^{k-1}, \mathbf{m}_{v}^{k})$$

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Graph Neural Networks With Attention

Graph attention networks ICLR 2018 GAT



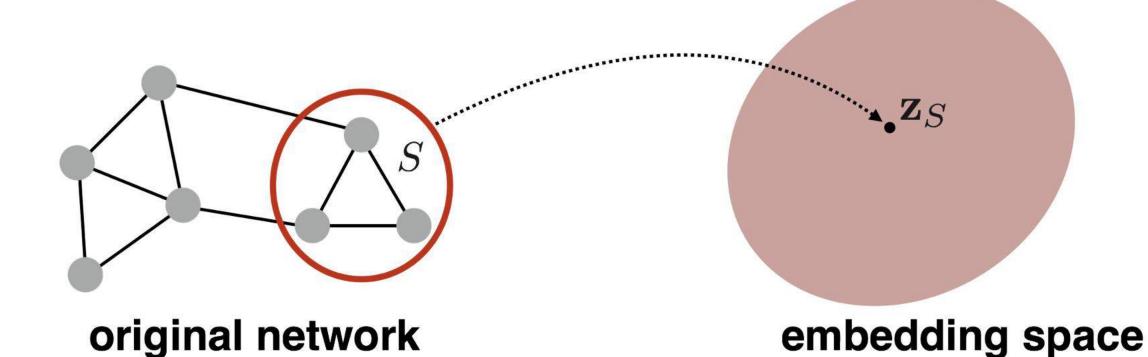
[Figure from Veličković et al. (ICLR 2018)]

Outline

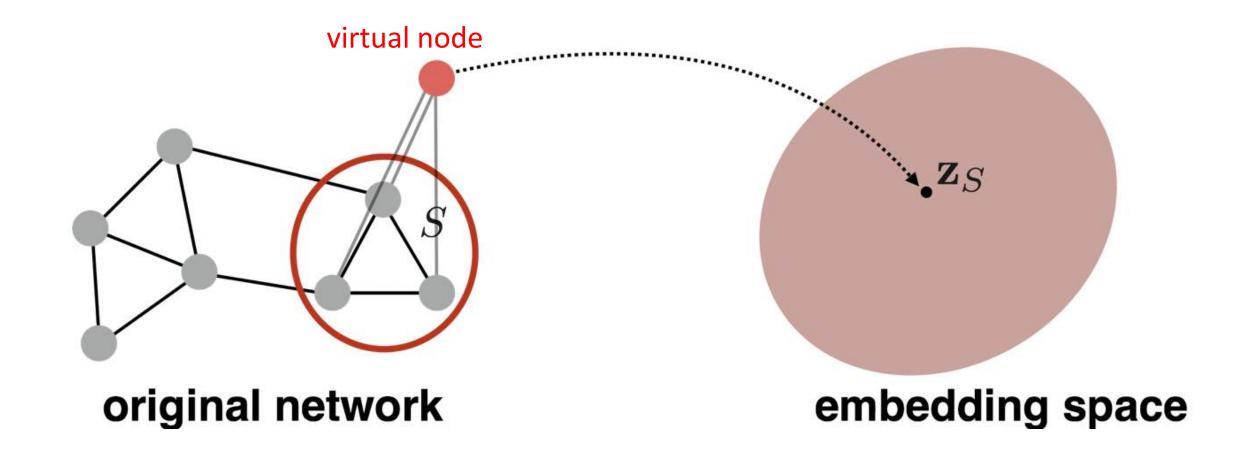
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Sub-Graph Embeddings

$$\mathbf{z}_S = \sum_{v \in S} \mathbf{z}_v$$



Sub-Graph Embeddings



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Message Passing Neural Network (MPNN)

- Unified various graph neural network and graph convolutional network approaches.
- A general framework for supervised learning on graphs.
- Two phases, a message passing phase and a readout phase.
- Message passing phase (namely, the propagation step)
 - Runs for *T* time steps
 - Defined in terms of message function M_t and vertex update function U_t .
- Readout phase
 - computes a feature vector for the whole graph using the readout function R

$$egin{aligned} \mathbf{m}_v^{t+1} &= \sum_{w \in \mathcal{N}_v} M_t\left(\mathbf{h}_v^t, \mathbf{h}_w^t, \mathbf{e}_{vw}
ight) \ \mathbf{h}_v^{t+1} &= U_t\left(\mathbf{h}_v^t, \mathbf{m}_v^{t+1}
ight) \end{aligned} \qquad \qquad \mathbf{\hat{y}} = R(\{\mathbf{h}_v^T ig| v \in G\})$$

MPNN && GGNN

$$M_t\left(\mathbf{h}_v^t, \mathbf{h}_w^t, \mathbf{e}_{vw}\right) = \mathbf{A}_{\mathbf{e}_{vw}} \mathbf{h}_w^t$$

$$U_t = GRU\left(\mathbf{h}_v^t, \mathbf{m}_v^{t+1}\right)$$

$$R = \sum_{v \in V} \sigma\left(i(\mathbf{h}_v^T, \mathbf{h}_v^0)\right) \odot\left(j(\mathbf{h}_v^T)\right)$$

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GNN IN NLP

AMR-To-Text

- A Graph-to-Sequence Model for AMR-to-Text Generation ACL 18
- Graph-to-Sequence Learning using Gated Graph Neural Networks ACL 18
- Structural Neural Encoders for AMR-to-text Generation NAACL 19

SQL-To-Text

SQL-to-Text Generation with Graph-to-Sequence Model EMNLP18

Document Summarization

- Structured Neural Summarization ICLR 19
- Graph-based Neural Multi-Document Summarization CoNLL 17

AMR

- Abstract Meaning Representation (AMR)
- **Graph**: rooted, directed graph
- nodes in the graph represent concepts and edges represent semantic relations between them

Task: recover a text representing the same meaning as an input AMR

graph.

Challenge

 word tenses and function words are abstracted away

Previous

- Seq2Seq Model
- linearized AMR structure
- **Problem**: closely-related nodes, such as parents, children and siblings can be far away after serialization.

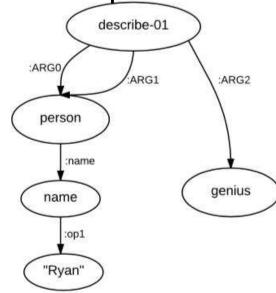


Figure 1: An example of AMR graph meaning "Ryan's description of himself: a genius."

Graph Encoder

$$G = (V, E)$$
 $g = \{h^j\}|_{v_j \in V}$

$$x_{j}^{i} = \sum_{(i,j,l) \in E_{in}(j)} x_{i,j}^{l}$$

$$x_{j}^{o} = \sum_{(j,k,l) \in E_{out}(j)} x_{j,k}^{l},$$

$$x_{j}^{i} = \sum_{(i,j,l)\in E_{in}(j)} x_{i,j}^{l}$$

$$x_{j}^{o} = \sum_{(j,k,l)\in E_{out}(j)} x_{j,k}^{l},$$

$$x_{i,j}^{l} = W_{4}([e_{l}; e_{i}]) + b_{4}$$

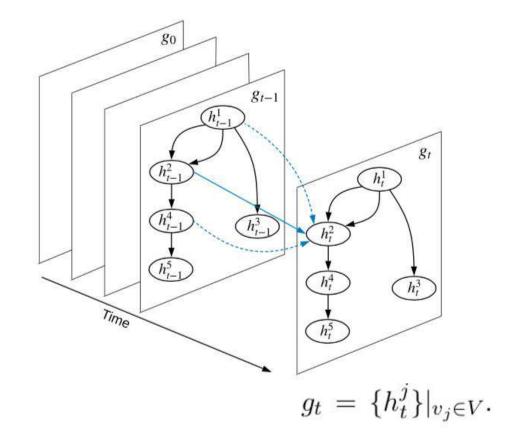
$$x_{i,j}^{l} = W_{4}([e_{l}; e_{i}; h_{i}^{c}]) + b_{4},$$

Node

Edge

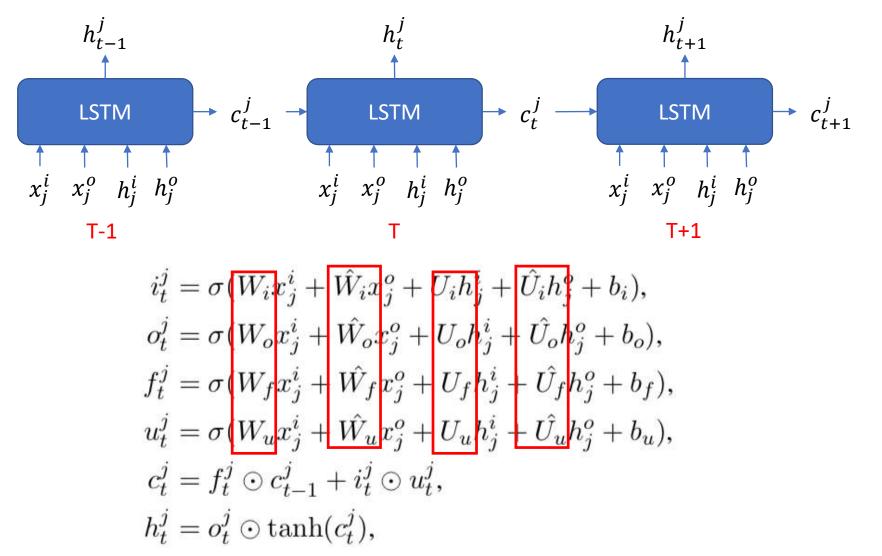
$$h_{j}^{i} = \sum_{(i,j,l) \in E_{in}(j)} h_{t-1}^{i}$$

$$h_{j}^{o} = \sum_{(j,k,l) \in E_{out}(j)} h_{t-1}^{k},$$

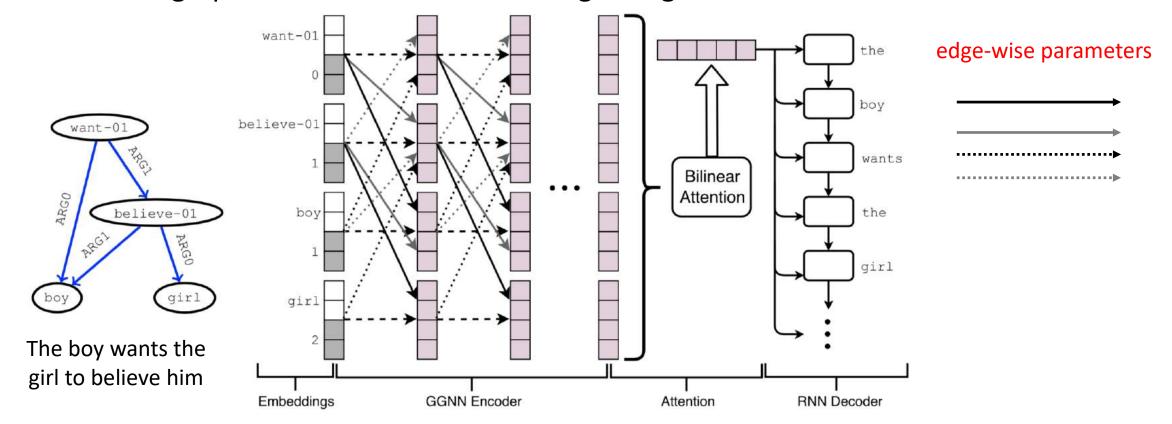


Graph Decoder

- Decoder initial state:average of the last states of all nodes.
- Each attention vector becomes $[h_T^j; x_j]$

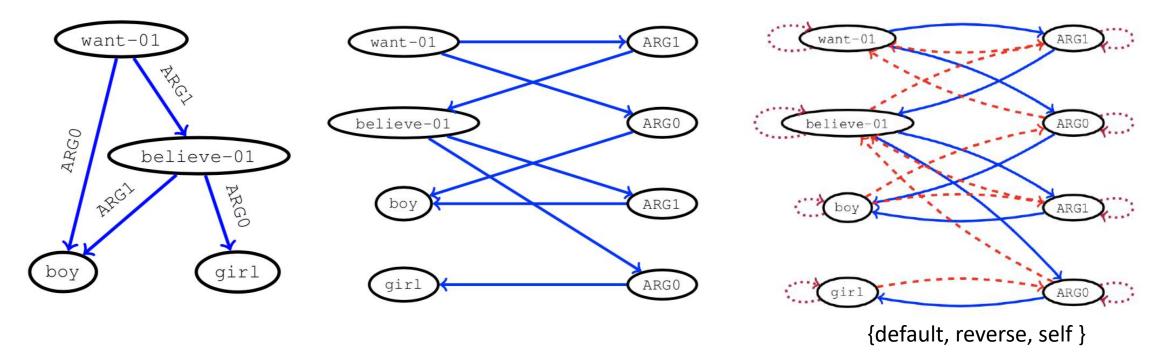


- Previous: represent edge information as label-wise parameters
- Nodes and edges to have their own hidden representations.
- Method: graph transformation that changes edges to additional nodes



Levi Graph Transformation

- Ideally, edges should have instance-specific hidden states
- Transform the input graph into its equivalent Levi graph



$$\begin{split} \mathcal{G} &= \{\mathcal{V}, \mathcal{E}, L_{\mathcal{V}}, L_{\mathcal{E}}\} \\ \mathbf{h}_v^0 &= \mathbf{x}_v \\ \text{reset} \quad \mathbf{r}_v^t &= \sigma \left(c_v^r \sum_{u \in \mathcal{N}_v} \mathbf{W}_{\ell_e}^r \mathbf{h}_u^{(t-1)} + \mathbf{b}_{\ell_e}^r \right) \\ \text{edge-wise parameters} \\ \text{update} \quad \mathbf{z}_v^t &= \sigma \left(c_v^z \sum_{u \in \mathcal{N}_v} \mathbf{W}_{\ell_e}^z \mathbf{h}_u^{(t-1)} + \mathbf{b}_{\ell_e}^z \right) \\ \widetilde{\mathbf{h}}_v^t &= \rho \left(c_v \sum_{u \in \mathcal{N}_v} \mathbf{W}_{\ell_e} \left(\mathbf{r}_u^t \odot \mathbf{h}_u^{(t-1)} \right) + \mathbf{b}_{\ell_e} \right) \\ \mathbf{h}_v^t &= (1 - \mathbf{z}_v^t) \odot \mathbf{h}_v^{(i-1)} + \mathbf{z}_v^t \odot \widetilde{\mathbf{h}}_v^t \end{split}$$

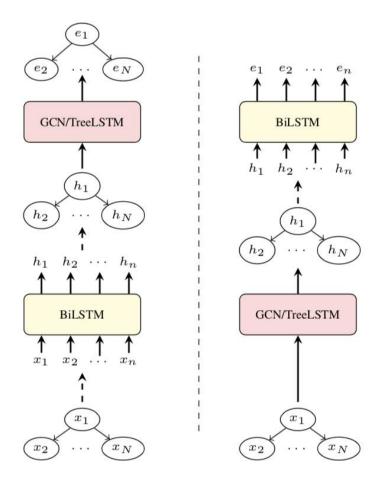
- Transforms the input graph into its equivalent Levi graph
- Graph Convolutional Network Encoders

$$h_i^{(k+1)} = \sigma \left(\sum_{j \in \mathcal{N}(i)} W_{\text{dir}(j,i)}^{(k)} h_j^{(k)} + b^{(k)} \right)$$

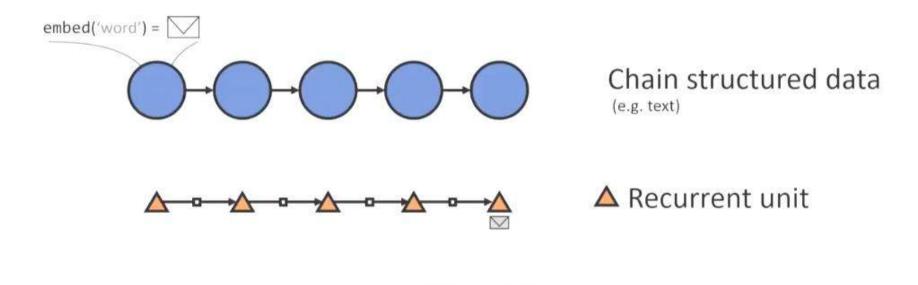
$$e_{1:N} = h_1^{(K)}, \dots, h_N^{(K)},$$

dir(j, i) indicates the direction of the edge between x_j and x_i

Stacking Encoders



- AMR is naturally a Graph.
- However, Text based NLP:



 $\square' = \bigwedge (\square, \square)$

GNN IN NLP

AMR-To-Text

- A Graph-to-Sequence Model for AMR-to-Text Generation ACL 18
- Graph-to-Sequence Learning using Gated Graph Neural Networks ACL 18
- Structural Neural Encoders for AMR-to-text Generation NAACL 19

SQL-To-Text

SQL-to-Text Generation with Graph-to-Sequence Model EMNLP18

Document Summarization

- Structured Neural Summarization ICLR 19
- Graph-based Neural Multi-Document Summarization CoNLL 17

SQL-to-Text

 SQL-to-text task is to automatically generate human-like descriptions interpreting the meaning of a given structured query language (SQL) query.

(SQL): **SELECT** company **WHERE** assets $> \text{val}_0$ **AND** sales $> \text{val}_0$

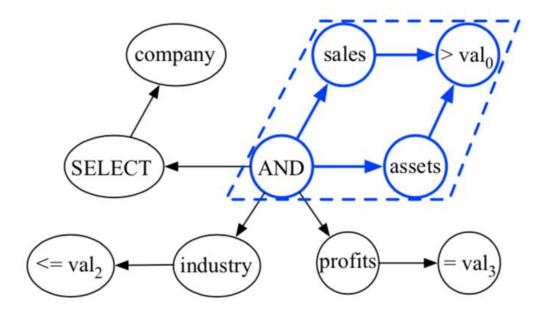
AND industry_rank <= val₂ **AND** revenue = val₃

Interpretation:

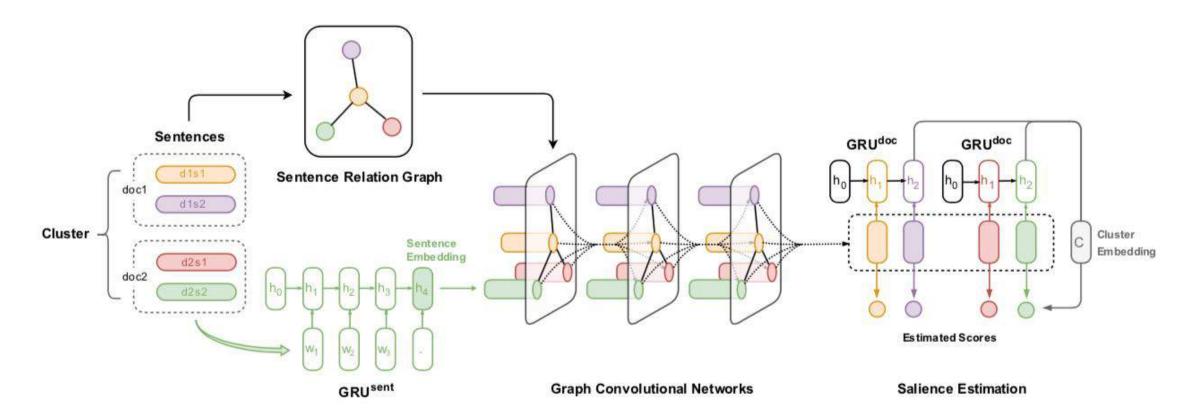
which company has both the market value and assets higher than val₀, ranking in top val₂ and revenue of val₃

SQL-to-Text

- **Motivation**: representing SQL as a graph instead of a sequence could help the model to better learn the correlation between this graph pattern and the interpretation "...both X and Y higher than Z..."
- SELECT Clause + WHERE Clause.



Task: Multi-Document Summarization(MDS)



Cosine similarity

- BoW: frequency based
- Threshold > 0.2
- TF-IDF First

Approximate Discourse Graph (ADG).

• The ADG constructs edges between sentences by counting discourse relation indicators such as deverbal noun references, event / entity continuations, discourse markers, and coreferent mentions. These features allow characterization of sentence relationships, rather than simply their similarity.

• Input

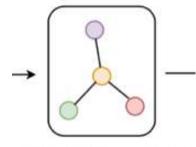
$$A \in \mathbb{R}^{N imes N}$$
 adjacency matrix $X \in \mathbb{R}^{N imes D}$ input node feature matrix

Output

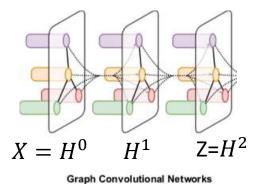
 $Z \in \mathbb{R}^{N imes F}$ high-level hidden features for each node

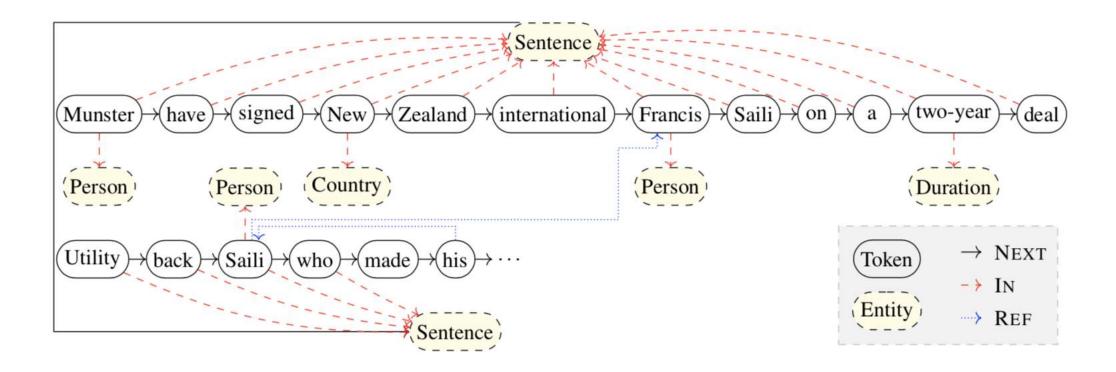
$$H^{(l+1)} = \sigma \left(AH^{(l)}W^{(l)} \right)$$

$$Z = f(X, A) = H^{(L)}$$

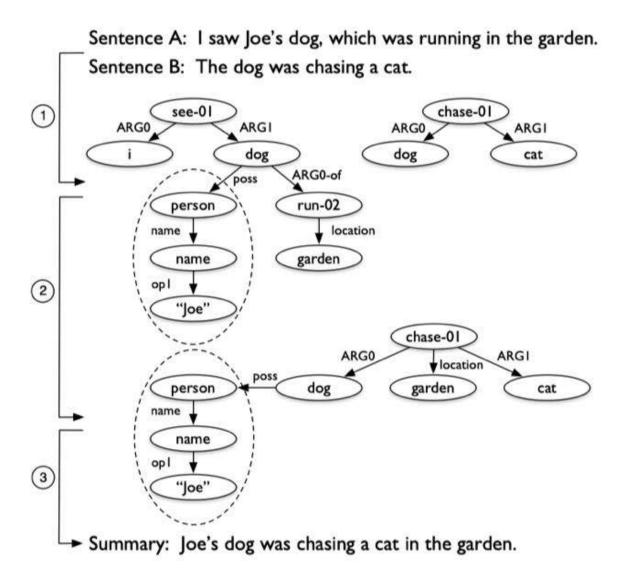


Sentence Relation Graph





Summarization && AMR

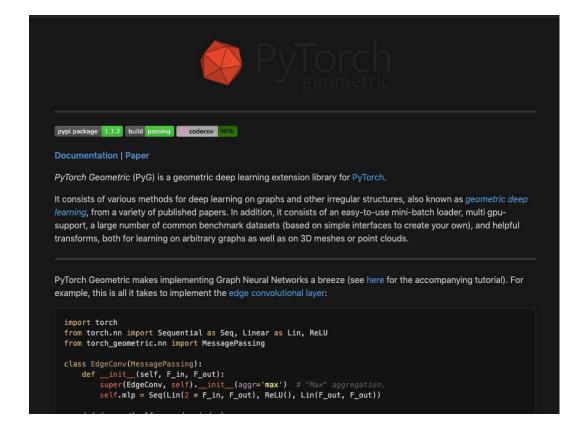


Outline

- 1. Basic && Overview
- 2. Graph Neural Networks
 - 1. Original Graph Neural Networks (GNNs)
 - 2. Graph Convolutional Networks (GCNs) && Graph SAGE
 - 3. Gated Graph Neural Networks (GGNNs)
 - 4. Graph Neural Networks With Attention (GAT)
 - 5. Sub-Graph Embeddings
- 3. Message Passing Neural Networks (MPNN)
- 4. GNN In NLP (AMR、SQL、Summarization)
- 5. Tools
- 6. Conclusion

Tools

- https://github.com/rusty1s/pytorch_geometric
- https://github.com/dmlc/dgl





Yann LeCun @ylecun · 2d

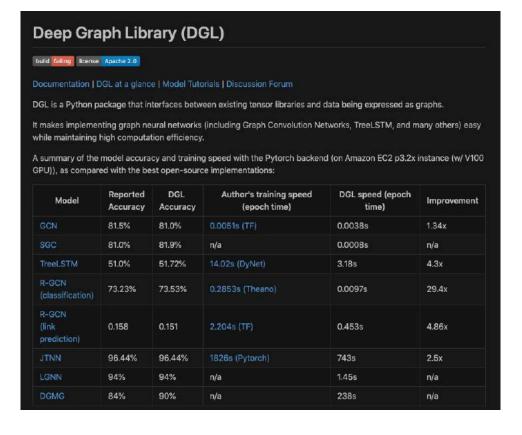
A fast & nice-looking PyTorch library for geometric deep learning (NN on graphs and other irregular structures).

Code: github.com/rusty1s/pytorc...
Paper: arxiv.org/abs/1903.02428

"Fast Graph Representation...



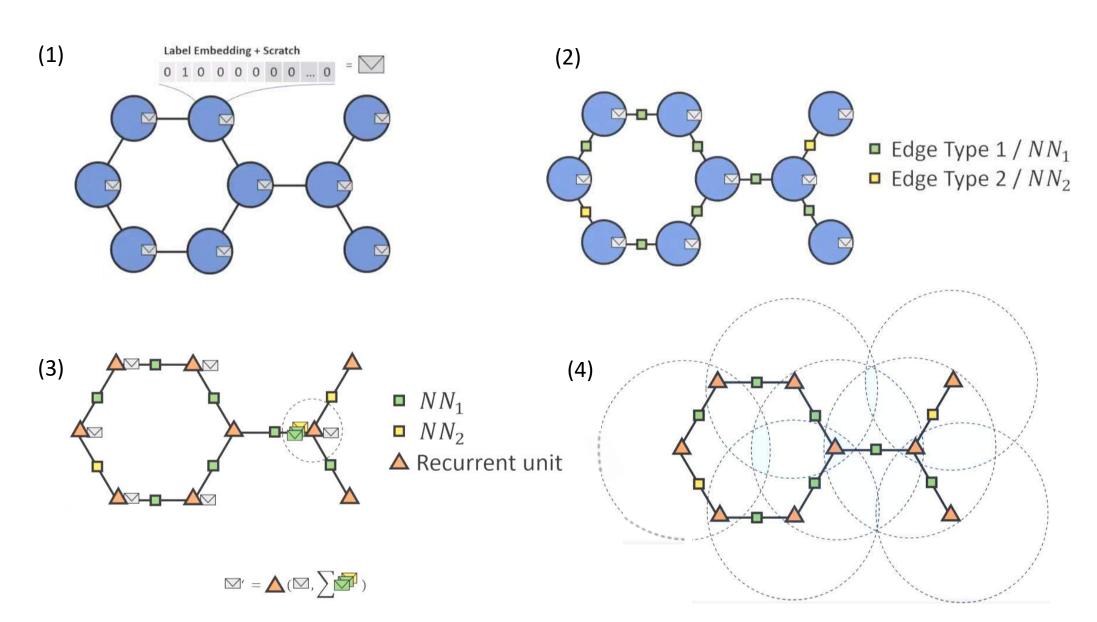
rusty1s/pytorch_geometric github.com



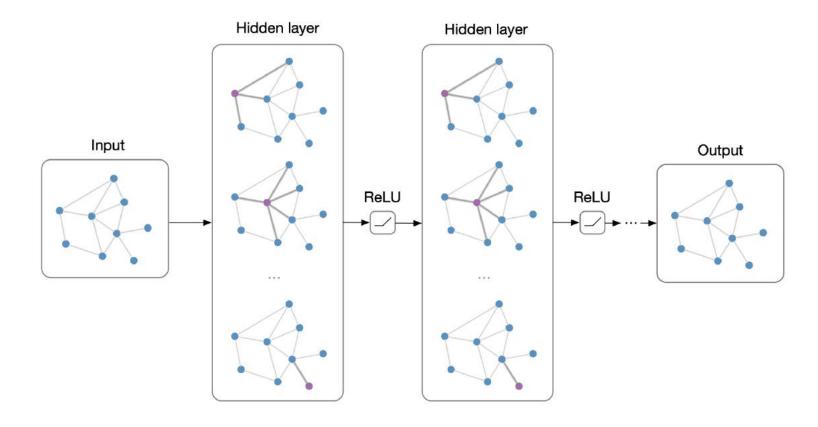
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Conclusion



Thanks!



Xiachong Feng TG 2019-04